NUP-A-2004-4

Remark on Natural Models of Neutrinos

Kazuo Fujikawa

Institute of Quantum Science, College of Science and Technology
Nihon University, Chiyoda-ku, Tokyo 101-8308, Japan

Abstract

We comment on what the naturalness argument of 't Hooft implies for a minimal extension of the standard model which incorporates right-handed neutrinos with generic mass terms. If this Lagrangian is taken as a low energy effective theory, the idea of pseudo-Dirac neutrinos with very small masses is consistent with the naturalness argument of 't Hooft. This argument is based on an observation that the right-handed components of neutrinos in the massless limit exhibit an extra enhanced symmetry which is absent in other charged fermions. This enhanced symmetry is reminiscent of the Nambu-Goldstone fermions associated with spontaneously broken supersymmetry. The conventional seesaw scenario gives another natural solution if the ultra-heavy right-handed neutrinos are integrated out in defining a low energy effective theory.

1 Introduction

It appears that no reliable theory of lepton and quark masses is known in the standard model [1], and thus the fermion masses and mixing angles are purely phenomenological parameters at this moment. The small neutrino masses indicated by the oscillation experiments [2, 3] could be even more deceptive than ordinary lepton and quark masses, and it may be worth examining the small neutrino masses from a general perspective independently of explicit detailed models.

In this note we comment on the general aspects of neutrino masses in the standard model with generic neutrino mass terms added by taking the naturalness argument of 't Hooft [4] as a guiding principle. At this moment, the popular picture of neutrinos with small observed masses appears to be the seesaw scenario [5] where the right-handed neutrinos with huge masses, when integrated out, induce small masses for the left-handed neutrinos in the low energy effective theory. The small lepton number violating terms in the low energy effective theory are thus natural in the sense of 't Hooft, since one recovers the lepton number conservation if one sets the small masses for the left-handed neutrinos to be zero. On the other hand, the Dirac neutrinos with small masses are consistent with oscillation experiments[2], but such neutrino mass terms appear to be neither generic nor

natural [1] . In this note we point out that almost Dirac-type neutrinos with tiny masses [2] are consistent with the naturalness argument of 't Hooft in the framework of a minimal extension of the standard model which contains right-handed neutrinos with generic mass terms. Our argument is based on an observation that the right-handed components of neutrinos exhibit an extra enhanced symmetry in the massless limit, which is absent in other charged fermions. This enhanced symmetry turns out to be reminiscent of the Nambu-Goldstone fermions [18, 19] associated with spontaneously broken supersymmetry.

2 One-generation model

We first study one-generation of leptons to explain the essence of the argument. We consider a minimal extension of the standard model [1] by incorporating the right-handed neutrino

$$\psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \psi_R = \begin{pmatrix} \nu_R \\ e_R \end{pmatrix} \tag{2.1}$$

and the part of the Lagrangian relevant to our discussion is given by

$$\begin{aligned} \mathcal{L} &= \overline{\psi}_L i\gamma^\mu(\partial_\mu - igT^aW^a_\mu - i\frac{1}{2}g'Y_LB_\mu)\psi_L \\ &\quad + \overline{\psi}_R i\gamma^\mu(\partial_\mu - i\frac{1}{2}g'Y_RB_\mu)\psi_R \\ &\quad + [-\overline{\psi}_R M\psi_L - \frac{1}{2}\nu_R^T C\mu\nu_R] + h.c. \end{aligned} \tag{2.2}$$

with $Y_L = -1$ and

$$Y_R = \begin{pmatrix} 0 & 0 \\ 0 & -2 \end{pmatrix}. \tag{2.3}$$

The Dirac mass term in the unitary gauge is given by

$$M = \begin{pmatrix} m_D + (m_D/v)\varphi & 0 \\ 0 & m_e + (m_e/v)\varphi \end{pmatrix} \tag{2.4}$$

where v stands for the vacuum value of the Higgs field, and the variable φ above stands for the Higgs field after subtracting the vacuum value. The operator C stands for the charge conjugation matrix for spinors.[3] The term with μ in the above Lagrangian is the Majorana mass term for the right-handed neutrino.

[1] From the view point of chiral symmetry, fermion masses are renormalized multiplicatively and thus any values of fermion masses may be said to satisfy the naturalness condition of 't Hooft. We search for other symmetries.

[2] These neutrinos are known generically as pseudo-Dirac neutrinos [6] - [13], and the possible dynamical schemes for pseudo-Dirac neutrinos have been analyzed from different view points in the past. See, for example, [14, 15, 16, 17] and references therein.

[3] We adopt the charge conjugation matrix convention

$$C\gamma^\mu C^{-1} = -(\gamma^\mu)^T, \quad C\gamma_5 C^{-1} = \gamma_5^T, \quad C^\dagger C = 1, \quad C^T = -C.$$

The above Lagrangian is formally the same as that for the conventional seesaw scenario if one chooses $\mu^2 \gg m_D^2$. The seesaw picture is mainly motivated by a grand unification idea such as $SO(10)$ [21]. In the following, we take the above Lagrangian as a *low energy effective theory* and apply the naturalness argument of 't Hooft to it. It is then argued that the choice $\mu^2 \ll m_D^2$ with m_D much smaller than the charged lepton mass is also natural in the low energy effective theory.[4]

The free part of the neutrino of this model is given by

$$\mathcal{L}_\nu = \bar{\nu} i\gamma^\mu \partial_\mu \nu - \bar{\nu} m_D \nu - (\frac{1}{2}\nu_R^T C \mu \nu_R + h.c.). \tag{2.5}$$

We here present an analysis of this simple model in some detail to set a notation for the realistic three-generation model in the next section. The mass terms are generic, and m_D and μ are chosen to be real by a suitable choice of the phase convention of field variables in the case of a single flavor. By defining $N_L^T C = \bar{\nu}_R$ (and consequently $\nu_R^T C = \bar{N}_L$), the above Lagrangian is written as

$$\mathcal{L}_\nu \;=\; \bar{\Psi}_L i\gamma^\mu \partial_\mu \Psi_L - \frac{1}{2}\Psi_L^T C \mathcal{M} \Psi_L + h.c. \tag{2.6}$$

for the neutrino field defined by

$$\Psi_L \;=\; \left(\begin{array}{c} \nu_L \\ N_L \end{array}\right) \tag{2.7}$$

with the mass matrix

$$\mathcal{M} \;=\; \left(\begin{array}{cc} 0 & m_D^T \\ m_D & \mu \end{array}\right), \tag{2.8}$$

though $m_D^T = m_D$ in the present single flavor case. We thus have the Majorana mass eigenvalues

$$m_\pm = \frac{1}{2}\mu \pm \sqrt{m_D^2 + (\frac{1}{2}\mu)^2} \tag{2.9}$$

and the mixing angle between ν_L and N_L, which defines an orthogonal transformation, is given by

$$\tan\theta = \frac{m_D}{\mu + \sqrt{m_D^2 + \mu^2}}. \tag{2.10}$$

The corresponding neutrino mass eigenstates are given by

$$\Psi_L \to \left(\begin{array}{c} \nu_{(1)L} \\ \nu_{(2)L} \end{array}\right) = \left(\begin{array}{c} \cos\theta \nu_L + \sin\theta N_L \\ -\sin\theta \nu_L + \cos\theta N_L \end{array}\right). \tag{2.11}$$

[4] The right-handed neutrino ν_R with a huge mass is not allowed to appear in low energy effective theory and thus it is integrated out in the seesaw scenario, while ν_R appears in the low energy effective theory itself in the present scheme.

The mixing angle θ is close to $\pi/4$ for the case of $m_D \gg \mu$. The Majorana mass eigenstates are then defined by

$$\Psi_M = \begin{pmatrix} \nu_{(1)L} + \nu_{(1)R} \\ \nu_{(2)L} + \nu_{(2)R} \end{pmatrix} \equiv \begin{pmatrix} \nu_{(1)M} \\ \nu_{(2)M} \end{pmatrix} \tag{2.12}$$

with $\nu_{(1)R} = [\bar{\nu}_{(1)L}C^{-1}]^T$ and $\nu_{(2)R} = [\bar{\nu}_{(2)L}C^{-1}]^T$, and the Lagrangian (2.5) is written as

$$\mathcal{L}_\nu = \frac{1}{2}\Psi_M^T C i\gamma^\mu \partial_\mu \Psi_M - \frac{1}{2}\Psi_M^T C \begin{pmatrix} m_+ & 0 \\ 0 & m_- \end{pmatrix} \Psi_M. \tag{2.13}$$

The minus sign in the neutrino mass is taken care of by a suitable chiral transformation. Effectively, the above may be regarded as a decomposition of a single Dirac neutrino, which is specified by ν_L and N_L, to a linear combination of two massive Majorana neutrinos.

The weak interaction is described by

$$\nu_L = \cos\theta\nu_{(1)L} - \sin\theta\nu_{(2)L} = (\frac{1-\gamma_5}{2})(\cos\theta\nu_{(1)M} - \sin\theta\nu_{(2)M}) \tag{2.14}$$

and the weak singlet state is given by

$$N_L = \sin\theta\nu_{(1)L} + \cos\theta\nu_{(2)L} = (\frac{1-\gamma_5}{2})(\sin\theta\nu_{(1)M} + \cos\theta\nu_{(2)M}). \tag{2.15}$$

The neutrino oscillation [24, 25, 9] in this case is similar to the (Dirac) neutrino rotation in a strong magnetic field[26][6]; the weak active left-handed state ν_L is rotated to a weak in-active right-handed state $N_L = [\bar{\nu}_R C^{-1}]^T$.

We now argue that

$$\mu^2 \ll m_D^2 \tag{2.16}$$

is consistent with the naturalness of 't Hooft. The basic postulate of the naturalness of 't Hooft is that a small parameter in low energy effective theory is natural only when one obtains an enhanced symmetry by setting such a small parameter to be 0 [4]. In the present case, if one sets $\mu = 0$ one recovers an extra enhanced symmetry, namely, the fermion number symmetry; this argument in the context of low-energy effective theory is a standard one in any analysis of pseudo-Dirac neutrinos [6]-[17]. We thus have the natural Majorana masses of the neutrinos as

$$m_\pm \simeq \frac{1}{2}\mu \pm m_D \tag{2.17}$$

where the negative mass is made positive by a suitable chiral transformation.

We next argue that the value of m_D which is much smaller than other charged lepton and quark masses is consistent with the naturalness of 't Hooft, since if one sets $m_D = 0$ in (2.4) (with $\mu = 0$) one finds an extra enhanced symmetry

$$\nu_R(x) \to \nu_R(x) + \eta_R \tag{2.18}$$

where η_R is a constant spinor, or in the Majorana notation

$$\psi_M(x) \to \psi_M(x) + \eta_M \tag{2.19}$$

where

$$\psi_M(x) = \nu_R(x) + \nu_L(x) \tag{2.20}$$

with $\nu_L = [\bar{\nu}_R C^{-1}]^T$. The existence of this special symmetry is a result of the fact that only ν_R is gauge singlet in the standard model. To adopt this simple symmetry as a basic symmetry of the effective theory, we need to assume that the enhanced symmetry (2.18) is a basic symmetry of the theory underlying the standard model. In more technical terms, we need to assume that the right-handed component of the neutrino couples to other heavier degrees of freedom in the effective Lagrangian, which do not explicitly appear in the standard model, either through the coupling which is proportional to the neutrino mass or through the derivative coupling which is suppressed in the low energy effective theory. In this note we assume that this is the case, though we have no convincing reason to assert why it should be so.

In the limit where all the Dirac-type masses vanish with the vacuum value v kept fixed (and with $\mu = 0$), all the fermions in the standard model become chirally symmetric. But only the right-handed component of the neutrino has the above extra stronger symmetry. Our assumption in this note is that this enhanced symmetry (2.18) is more effective for the tiny neutrino mass than the chiral symmetry which affects all the fermion masses universally.

Our analysis so far has no connection with supersymmetry. However, the symmetry (2.18) where the fermion field transforms inhomogeneously with a constant component is reminiscent of a Nambu-Goldstone fermion in spontaneously broken supersymmetric theory [27]. Since the Nambu-Goldstone fermion implies a tiny mass, it may be tempting to entertain the idea of possible connection of the enhanced symmetry (2.18) with supersymmetry. We however note that the Nambu-Goldstone fermions satisfy the enhanced symmetry (2.18) in low energy scattering amplitudes [27] (and thus in the low energy efffective action), but the enhanced symmetry (2.18) by itself does not necessarily imply the existence of spontaneously broken supersymmetry in the deep level. [5]

It may be interesting to recall that the idea of the neutrino as a Nambu-Goldstone fermion was suggested immediately after the discovery of supersymmetry [18, 19], but the idea has been later abandoned since the (left-handed) neutrino in the Fermi interaction does not decouple in the low-energy limit contrary to the basic property expected for a

[5] If our speculation on the possible connection with supersymmetry should be valid, the transformation law (2.18) would be replaced by

$$\nu_R(x) \to \nu_R(x) + \eta_R + O_R(x)$$

in a full renormalizable supersymmetric theory, where $O_R(x)$ stands for the fields or composite operators representing the heavy degrees of freedom which consist of superpartners. The observed tiny neutrino mass and the absence of supersymmetric particles in the standard model suggest that (presumed) supersymmetry is explicitly broken in a very specific way such that the notion of Nambu-Goldstone fermions is not completely spoiled.

Nambu-Goldstone particle [20]. In contrast, the right-handed component of the neutrino in fact decouples from the Fermi interaction in the low energy limit.[6]

In any case, the present naturalness argument which is based on the enhanced special symmetry (2.18) for the right-handed neutrino does not contradict the observed fact that the neutrino masses are very small compared to other charged fermion masses, regardless of whether the above enhanced symmetry is possibly associated with supersymmetry or not.

3 Three-generation model

We now discuss a realistic three-generation model where three right-handed neutrinos with generic mass terms are added to the standard model. Our formulas (2.1) and (2.2) are valid for the case of three generations of fermions if one understands that $e_{L,R}(x)$ there contains 3 components as

$$e_{L,R}(x) \quad \rightarrow \quad \begin{pmatrix} e_{L,R}(x) \\ \mu_{L,R}(x) \\ \tau_{L,R}(x) \end{pmatrix} \tag{3.1}$$

and, correspondingly, the neutrino fields ν_L and ν_R respectively contain 3 fields. The neutrino field in (2.7) is then replaced by

$$\Psi_L \quad = \quad \begin{pmatrix} \nu_L \\ N_L \end{pmatrix} \tag{3.2}$$

where

$$\nu_L \quad = \quad \begin{pmatrix} \nu_L^{(1)} \\ \nu_L^{(2)} \\ \nu_L^{(3)} \end{pmatrix}, \quad N_L = \begin{pmatrix} N_L^{(1)} \\ N_L^{(2)} \\ N_L^{(3)} \end{pmatrix} = \begin{pmatrix} [\bar{\nu}_R^{(1)} C^{-1}]^T \\ [\bar{\nu}_R^{(2)} C^{-1}]^T \\ [\bar{\nu}_R^{(3)} C^{-1}]^T \end{pmatrix}. \tag{3.3}$$

The extra fields $N_L^{(1)} \sim N_L^{(3)}$ stand for the gauge singlet neutrinos. The mass matrix of the neutrinos in (2.8) is replaced by

$$\mathcal{M} \quad = \quad \begin{pmatrix} 0 & m_D^T \\ m_D & \mu \end{pmatrix} \tag{3.4}$$

where m_D and μ now stand for 3×3 matrices of complex numbers in general, and μ is a symmetric matrix. The matrix $\mathcal{M}$ thus contains 15 complex parameters in general.

In the present three-generation case also, the naturalness argument of 't Hooft in low energy effective theory is consistent with[7] $||\mu|| \ll ||m_D||$ because of the breaking of lepton

[6]The right-handed component of the neutrino in the massless limit simply decouples from all the interactions in the standard model. The decoupling of the right-handed neutrino from possible heavy superpartners in low energy effective theory is at least consistent with the suggested Nambu-Goldstone nature of the right-handed neutrino.

[7]The notation $||\mu|| \ll ||m_D||$ may be understood as meaning that the largest eigenvalue of μ is much smaller than the smallest eigenvalue of m_D.

number symmetry by the Majorana mass μ. A further naturalness argument on the basis of a generalization of the enhanced symmetry (2.18) for right-handed neutrinos, which appears if one sets $m_D = 0$ (with $\mu = 0$), is also consistent with the Dirac-type neutrino masses m_D which are much smaller than the masses of other charged leptons and quarks. We are of course assuming that the enhanced symmetry (2.18), due to the reasons not understood at this moment, is more effective for ensuring the tiny neutrino masses than the chiral symmetry which appears universally for all the fermions when the Dirac-type masses are set to zero. To be more explicit, we have a generalization of the enhanced symmetry (2.19) for $m_D = 0$ (with $\mu = 0$),

$$\psi_M^{(i)}(x) \rightarrow \psi_M^{(i)}(x) + \eta_M^{(i)}, \qquad i = e, \mu, \tau \tag{3.5}$$

with $\psi_M^{(i)}(x) = \nu_R^{(i)}(x) + [\bar{\nu}_R^{(i)}(x) C^{-1}]^T$. The existence of this special symmetry is a result of the fact that only ν_R is gauge singlet in the standard model. Our basic assumption is that the above enhanced symmetry (3.5) is a basic symmetry of the theory underlying the standard model.

Our analysis so far is independent of supersymmetry or any other fermionic symmetry. However, the enhanced symmetry (3.5) is suggestive of the Nambu-Goldstone nature of the right-handed neutrinos, and it is tempting to entertain the idea that this enhanced symmetry is associated with supersymmetry. But we have extra complications in the attempt of this interpretation in the three-generation case. First of all, the number of Nambu-Goldstone fermions agrees with the number of generators of spontaneously broken supersymmetry. The three Nambu-Goldstone fermions thus suggest an extended $N = 3$ supersymmetry with three Majorana-type supercharges. The possible association of three generations of fermions with $N = 3$ generators of supersymmetry is interesting, but the extended $N = 3$ supersymmetry introduces a plethora of exotic particles in the full theory [27]. Also, the interplay between the explicit and spontaneous breakings of possible supersymmetry becomes more involved.

However, we would like to indicate that the above possible association with supersymmetry does not lead to an outright contradiction, by recalling a model [22] where all the leptons and quarks in the standard model are understood as Nambu-Goldstone fermions arising from spontaneously broken supersymmetry. In their scheme, the finite masses of leptons and quarks are mainly attributed to the explicit supersymmetry breaking by gauge interactions appearing in the standard model. In this interpretation, the neutrino masses are expected to be special since the right-handed components are gauge singlet. In our naturalness argument above, the enhanced symmetry (3.5) also arises from the fact that the right-handed neutrinos are gauge singlet. Their model may show that the possible association of the enhanced symmetry (3.5) with supersymmetry does not lead to an outright contradiction, though our argument here is not identical to their explicit model. It may be interesting to investigate the model in [22] further in view of our observation of the enhanced symmetry.

We consider that our argument on the basis of the naturalness of 't Hooft for the small Dirac-type neutrino masses m_D is valid to the extent that the above enhanced symmetry (3.5) is taken to be a basic symmetry of the theory underlying the standard model,

quite independently of the possible association of the enhanced symmetry (3.5) with supersymmetry. We briefly comment on the phenomenological implications of pseudo-Dirac neutrinos in the following with the proviso that this is the case.

It is known that there are 12 independent *complex* parameters to characterize the lepton mixing [28] in the present case. In terms of these 12 complex parameters together with 6 real Majorana-type mass parameters (which comprise 15 complex parameters), we have the following main physical properties to be analyzed:
1. Neutrino oscillation
2. Neutrinoless double β decay
3. Magnetic moment
4. CP violation
In the most general case, the analyses of these properties are quite involved. If one assumes the limit $||\mu|| \ll ||m_D||$ as in our case, the analyses become slightly easier, but the inter-connection of the above properties is still complicated and interesting.

There exist the detailed analyses of the experimental implications of pseudo-Dirac neutrinos in the literature, for example, in [23] and we here briefly comment on some limiting cases. We classify the possible cases into several categories:
(i) The pure Dirac case $\mu = 0$
In this case, the neutrino oscillation is just a standard one[24, 25], and no neutrinoless double β decay[29]. The magnetic moment is also the standard one[26], and the CP violation is described by a copy of the KM scheme in the quark sector[30].

(ii) Generic Dirac mass term m_D with $||m_D|| \gg ||\mu||$
In this case, the mass splitting of neutrinos are mainly controlled by the mass eigenvalues of m_D. The mass spectrum indicated by oscillation experiments is basically specified by m_D, since oscillation experiments indicate that the oscillation into "sterile" components N_L is small [2, 3]. The major structure of the neutrino mass spectrum is determined by m_D, and each mass eigenstate of m_D is modulated by the perturbation of μ as in the case of the single generation model. The double β decay, which is induced by μ, is thus much suppressed compared to what one expects for purely Majorana fermions such as in the seesaw scenario. The CP violation is also basically controlled by the mass matrix m_D and thus similar to the case of quarks. The mass eigenstates of all the neutrinos are Majorana-type in a strict sense for $\mu \neq 0$, and thus the magnetic moments of neutrinos are basically transitional ones[31]. However, if the magnitudes of the magnetic moments are small as is indicated by the calculation in the standard model[26], the magnetic transition is mainly intra-flavor transitions. For example, the electron-type neutrino stays electron-type, though the magnetic transition may cause a transition from one electron-type Majorana neutrino to another electron-type Majorana neutrino. In this sense, the magnetic transition is similar to the case of the Dirac neutrinos (and the physical effects of the magnetic transition may be rather minor compared to the oscillation). If the magnetic moment is large as is allowed by a phenomenological analysis of the presently available experimental limit and if the magnetic field is strong enough (such as in the neighborhood of some of the neutron stars)[26], the magnetic transition can cause the inter-flavor transitions. In such a case, the interplay of the magnetic transition and the

oscillation can cause interesting observable phenomena[32].

(iii) Degenerate Dirac mass matrix m_D with $||m_D|| \gg ||\mu||$
It is interesting to examine a specific limiting case where the Dirac mass matrix after diagonalization has eigenvalues such as

$$m_D = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{3.6}$$

with $m_1 > m_3$.

The CP violation in this limiting model does not arise from the mass matrix m_D (since we are effectively dealing with a two-generation model) and thus CP violation entirely comes from the CP violation in the lepton number violating Majorana mass term μ. However, the effects of CP violation may not necessarily be small in the present degenerate case. Also the mass splitting among the heavier neutrinos measured by oscillation experiments faithfully indicates the magnitude of the Majorana mass term μ, and thus the double β dacay can take place at the rate estimated on the basis of the ν_e oscillation experiments.

To be more specific, we treat the Majorana mass term proportional to μ as a small perturbation. Then the diagonalization of the Dirac mass by neglecting the Majorana mass for the moment gives rise to the mixing matrix of the left-handed neutrinos coupled to the charged weak current as

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \tag{3.7}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. We used the degeneracy of ν_1 and ν_2 to define that only ν_2 to be mixed with ν_3. In this procedure the Majorana mass μ for the right-handed neutrinos is generally replaced by a symmetric $\tilde{\mu}$. By recalling our assumption $||m_D|| \gg ||\tilde{\mu}|| \sim ||\mu||$, we consider that the isolated ν_3 is not much influenced by the Majorana mass term $\tilde{\mu}$. We thus retain only the first 2×2 components of $\tilde{\mu}$ and analyze their effects on the degenerate ν_1 and ν_2; we let ν_3 remain a Dirac neutrino in this procedure. The symmetric complex 2×2 matrix $\tilde{\mu}$, which contains 6 real parameters, can be diagonalized by a 2×2 unitary matrix as[28]

$$\tilde{\mu} = u^T \begin{pmatrix} \mu_1 & 0 \\ 0 & \mu_2 \end{pmatrix} u, \tag{3.8}$$

where u is a 2×2 unitary matrix which contains 4 real parameters.

We thus have the (approximate) neutrino mixing matrix for the charged current

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \begin{pmatrix} u^\dagger & 0 \\ 0 & 1 \end{pmatrix} \tag{3.9}$$

where the three neutrinos are still treated as Dirac neutrinos. One can confirm that the overall phase of u is eliminated and thus u becomes $SU(2)$ matrix.

The mass matrix of the first two generations of neutrinos, which contain 4 Majorana neutrinos, is then approximately given by

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_1 \\ m_1 & 0 & \mu_1 & 0 \\ 0 & m_1 & 0 & \mu_2 \end{pmatrix}. \tag{3.10}$$

This mass matrix is diagonalized by a further orthogonal $O(4)$ matrix, and the mass eigenvalues for 4 Majorana neutrinos are given by

$$\lambda \simeq m_1 \pm \frac{\mu_1}{2}, \quad m_1 \pm \frac{\mu_2}{2} \tag{3.11}$$

after a suitable chiral transformation to make all the masses positive. The orthogonal $O(4)$ matrix corresponds to the $O(2)$ mixing matrix in (2.11).

A salient feature of this limiting case is that both of the oscillation and double β decay are controlled by the Majorana mass term μ. The experiments indicate that the transition to the "sterile" components N_L is small, and the oscillation among different flavors is dominant [2, 3]. In the present limiting case, the oscillation between different flavors and the oscillation between the active and "sterile" neutrinos are expected to be comparable as is indicated by the mass formula (3.10) and (3.11). This suggests that the present limiting case (which is sometimes referred to as inverted hierarchy), though theoretically interesting, is not favored by experiments.

In the present scheme of three pseudo-Dirac neutrinos, the generic case (ii) appears to be most favored by experiments. See also [23].

4 Discussion

The Dirac-type neutrinos with tiny masses, as suggested by J. Steinberger [33] among others, are interesting but may appear to be neither generic nor natural. In the present note, we started with the generic mass terms for three generations of leptons in a minimal extension of the standard model and we first argued that the neutrino mass matrix which is close to the Dirac mass, namely $\mu^2 \ll m_D^2$, is consistent with the naturalness argument of 't Hooft for a low energy effective theory. By applying a further argument of the naturalness on the basis of the enhanced symmetry (3.5), we argued that the Dirac-type masses m_D of the neutrinos which are much smaller than other lepton and quark masses in the standard model are natural. We also entertained the idea that this enhanced symmetry may possibly be related to supersymmetry in the deep level. Starting with the Lagrangian which has apparently the same form as in the seesaw model, we thus identify two completely different natural models of neutrinos depending on how to understand the mass terms of right-handed neutrinos in low energy effective theory.

The naturalness argument as such cannot be water-tight, but the naturalness is useful in helping to guess the plausible dynamics behind the experimentally observed facts.

Our naturalness argument, though may not yet be a convincing one, suggests a possible association of the tiny neutrino masses with the interesting idea of Nambu-Goldstone fermions [18, 19]. Quite apart from the present analysis, it may be natural to expect that supersymmetry plays an intrinsic role in understanding the observed small neutrino masses, [8] if supersymmetry should be realized in nature at all,

I thank R. Shrock for a comment at the early stage of the present study.

References

[1] S. Weinberg, Phys. Rev. Lett. **19**, 1264 (1967).
A. Salam, in *Elementary Particle Theory*, edited by N. Svartholm (Stockholm, 1968) p.367.
S.L. Glashow, Nucl. Phys. **22**, 579 (1961).

[2] S. Pakvasa and J.W.F. Valle, "Neutrino properties before and after KamLAND", Proc. Indian Natl. Sci. Acad. **70A**, 189 (2004), and references therein.

[3] A.Y. Smirnov, "Neutrino physics: Open theoretical questions", Int. J. Mod. Phys. **A19**, 1180 (2004), and references therein.

[4] G. 't Hooft, "Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking", in *Recent Developments in Gauge Theories*, Cargese 1979, eds. G. 't Hooft et al., (Plenum, New York, 1990).

[5] T. Yanagida, in Proceedings of Workshop on Unified Theory and Baryon Number in the Universe, ed. by O. Sawada and A. Sugamoto (KEK report 79-18, 1979) p. 95.
M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, ed. by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979), p. 315.

[6] L. Wolfenstein, Nucl. Phys. **B186**, 147 (1981).

[7] S.T. Petcov, Phys. Lett. **B110**, 245(1982).

[8] S.M. Bilenky and B. Pontecorvo, Sov. J. Nucl. Phys. **38**, 248 (1983).

[9] S.M. Bilenky and S.T. Petcov, Rev. Mod. Phys. **59**, 671 (1987), and references therein.

[8] The neutrino masses in connection with the R-symmetry in supersymmetric theory have been analyzed in the past [34, 35, 36, 2, 3]. The properties of the theory with spontaneously broken supersymmetry are known to be very restrictive [27], and it may be interesting to examine if a concrete model, which incorporates the possible association of tiny neutrino masses with Nambu-Goldstone fermions, is constructed. The idea such as supersymmetric D-branes or other dynamical schemes may play a central role. The spontaneous breaking of internal symmetry in supersymmetric solitons and the quasi Nambu-Goldstone fermions [37] appearing there have been analyzed in [38], for example, but to our knowledge the modern analysis of Nambu-Goldstone fermions themselves with model building in mind appears to be missing except for the attempt in [22].

[10] C. Giunti, C.W. Kim and U.W. Lee, Phys. Rev. **D46**, 3034 (1992).

[11] J.P. Bowes and R.R. Volkas, J. Phys. **G24**, 1249 (1998).

[12] K.R. Balaji, A. Kalliomaki and J. Maalampi, Phys. Lett. **B524**, 153 (2002).

[13] M. Kobayashi and C.S. Lim, Phys. Rev. **D64**, 013003 (2001).

[14] P. Langacker, Phys. Rev. **D58**, 093017 (1998).

[15] G. Cleaver, M. Cvetic, J.R. Espinosa, L. Everett, and P. Langacker, Phys. Rev. **D57**, 2701 (1998)

[16] D. Chang and O.C. Kong, Phys. Lett. **B477**, 416 (2000).

[17] K.R. Dienes, E. Dunes and T. Gherghetta, Nucl. Phys. **B557**, 25 (1999).
G.R. Devali and A.Y. Smirnov, Nucl. Phys. **B563**, 63 (1999).
N. Arkani-Hamed, S. Dimopoulos, G.R. Dvali and J. March-Russel, Phys. Rev. **D65**, 024032 (2002).

[18] D.V. Volkov and V.P. Akulov, Phys. Lett. **B46**, 109 (1973).

[19] A. Salam and J. Strathdee, Phys. Lett. **B49**, 465 (1974).

[20] B. de Wit and D.Z. Freedman, Phys. Rev. Lett. **35**, 827 (1975).

[21] M. Fukugita and T. Yanagida, *Physics of Neutrinos and Applications to Astrophysics* (Springer-Verlag, Berlin Heidelberg, 2003).

[22] W.A. Bardeen and V. Visnjic, Nucl. Phys. **B194**, 422 (1982).

[23] J.F. Beacom, N.F. Bell, D. Hooper, J.G. Learned, S. Pakvasa and T.J. Weiler, Phys. Rev. Lett. **92**, 011101 (2004).

[24] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[25] V. Gribov and B. Pontecorvo, Phys. Lett. **B28**, 463 (1969).

[26] K. Fujikawa and R. Shrock, Phys. Rev. Lett. **45**, 963 (1980).

[27] J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton Univ. Press, 1992).
S. Weinberg, *The Quantum Theory of Fields*, Vol.III (Cambridge Univ. Press, 2000).

[28] J. Schechter and J.W.F. Valle, Phys. Rev. **D22**, 2227 (1980).

[29] M. Doi, T. Kotani, H. Nishiura, K. Okuda and E. Takasugi, Phys. Lett. **102B**, 323 (1981), and references therein.

[30] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[31] J.F. Nieves, Phys. Rev. **D26**, 3152 (1982).

[32] C.S. Lim and W.J. Marciano, Phys. Rev. **D37**, 1368 (1988).

[33] J. Steinberger, as is quoted in S.L. Glashow, " Fact and fancy in neutrino physics. 2", hep-ph/0306100.

[34] C. Aulakh and R. Mohapatra, Phys. Lett. **B119**, 136 (1983).

[35] L. Hall and M. Suzuki, Nucl. Phys. **B231**, 419 (1984).

[36] Y. Chikashige, R. Mohapatra and R. Peccei, Phys. Lett. **B98**, 265 (1980).

[37] W. Buchmuller, S.T. Love, R.D. Peccei and T. Yanagida, Phys. Lett. **B115**, 233 (1982).
W. Buchmuller, R.D. Peccei and T. Yanagida, Phys. Lett. **B124**, 67 (1983).

[38] M. Eto, M. Nitta and N. Sakai, "Effective theory on non-Abelian vortices in six dimensions ", hep-th/0405161, and references therein.